Use of Dynamic Course Core to Develop Teachers' Subject Matter Knowledge of Mathematics

Wei-Chi YANG
wyang@radford.edu
Department of Mathematics and Statistics
Radford University
Radford, VA 24142
USA

Agida G. MANIZADE
amanizade@radford.edu
Department of Mathematics and Statistics
Radford University
Radford, VA 24142
USA

Abstract

In this paper, we propose that dynamic course core has an essential role in developing teacher's subject matter knowledge of mathematics. We are presenting three technology based mathematical conceptual activities designed to improve teachers' mathematical knowledge for teaching the mean value theorem, the Cauchy mean value theorem, the inverse image of parametric curves, and the global minimum of total squared distances among non-intersecting curves or surfaces. These activities can help to develop teachers' representational fluency of aforementioned topics. They can also deepen teachers' conceptual understanding of these mathematical constructs through the use of technological tools in teachers' education and mathematics instruction.

1 Introduction

As mathematics educators in any part of the world, we would like to see mathematics teachers who posses the following qualifications:

1. have strong professionally situated knowledge, such as mathematical knowledge for teaching;
2. competent in their procedural and conceptual knowledge of mathematics; and

3. present evidence of professional competence to promote and scaffold this knowledge among their students.

In this paper, we address how these goals can be archived through the innovative use of technological tools in teachers' education as well as in school mathematics instructions. Readers are presented with three specific technology based activities that address the following mathematical topics: the mean value theorem, the Cauchy mean value theorem, the inverse image of parametric curves, and the global minimum of total squared distances among non-intersecting curves or surfaces. We propose these types of activities should be included as part of dynamic core of a mathematics curriculum.

2 Developing Mathematical Knowledge for Teaching

Mathematics educators developed a construct of teachers' **mathematical knowledge for teaching (MKT)**, that consists of:

- common content knowledge (CCK) defined as mathematical knowledge used in settings outside of teaching,
- specialized content knowledge (SCK) defined as mathematical knowledge and skills used in teaching,
- knowledge at the mathematical horizon (KMH) defined as knowledge of how mathematical topics relate to other mathematics ideas in the curriculum,
- knowledge of content and students (KCS) defined as a type of knowledge intertwined with knowledge of how students think, know, or learn a given topic,
- knowledge of content and teaching (KCT) defined as ability to make decisions on sequencing of activities, awareness of possible advantages and disadvantages of representations used, and ability to conduct quality classroom discourse, and
- knowledge of curriculum (KC) (see [1,8]).

MKT was developed based on Lee Shulman's original construct of teachers' subject matter knowledge (SMK) and pedagogical content knowledge (PCK) ([12,13]). In this paper, we are presenting ways where teachers can increase their CCK, SCK and KMH (constructs that correspond to Shulman's SMK) through the integration of Dynamic Geometry Software and Computer Algebra System.

We propose that a course containing both static and dynamic cores can help teachers to develop their subject matter knowledge. We define *static core (SC)* as the essence of a course based on the identified mathematical goals and objectives, and describe *dynamic core (DC) of a course* as part that includes opportunities for new discoveries using available technological tools. We think that there is a relationship between both static and dynamic cores. It reminds us the bicycle mechanism of two wheels connected by a chain. Affect of the DC can bring a type of change in mathematics classroom discourse that leads to learners' development of conceptual knowledge of mathematics (achieved by constructing the relationships between pieces of information in mathematics), and preservice teachers' development of SCK and KMH. In addition, this affect of dynamic core brings 'motion' to a static core that can be responsible for development of learners' procedural knowledge (composed of symbol representation system of mathematics, and algorithms, or rules for completing mathematical tasks) of mathematics and preservice teachers' CCK of mathematics. On the other hand, the implementation of static core can transform a classroom discourse in a way that might affect the quality of learners' engagement with dynamic core. As learners develop more procedural knowledge and teachers develop more common content knowledge, they can continue to acquire conceptual knowledge, specialized content knowledge, and horizon content knowledge of mathematics.

We suggest that technology can be used to develop teachers' CCK in section 3.1 and SCK by presenting two examples in section 3.2; more specifically, as an essential part of growth in teachers' CCK and SCK, we consider their development of conceptual knowledge and procedural knowledge of mathematics that are linked to the conceptual knowledge ([5,6,7]). Haapasalo and Kadijevich provided an in-depth critical analysis of the literature on conceptual and procedural knowledge ([7]). They described conceptual knowledge as being dynamic in nature and synthesized the following distinction:

- "Procedural knowledge denotes dynamic and successful utilization of particular rules, algorithms or procedures within relevant representation forms(s). ... it often calls for automated and unconscious steps.
- Conceptual knowledge denotes knowledge of a skilful drive along particular networks, the elements of which can be concepts, rules..., and even problems... given in various representation forms... it typically requires conscious thinking." ([7]).

Researchers farther provide critical analysis of the different views, such as inactivation view, simultaneous activation view, dynamic interaction view, generic view, of relationships of procedural and conceptual knowledge discussed in the education literature ([6,7]). We believe that conceptual knowledge may have a greater influence on procedural knowledge than the reverse. In other words, if time is an essence, we chose to focus on development of conceptual knowledge prior to development of procedural knowledge. Haapasalo states that "it seems appropriate to claim that the goal of any education should be to invest in conceptual knowledge from very beginning." ([6]). As researchers confirm this approach has been accepted by the international community of mathematics educators in many countries ([5,6]). In addition, several non-academic reports support the aforementioned approach. In 2001, as reported by the Beijing Youth Daily, the standards of National Curriculum in China have indicated that: a) "fill the duck" (rote learning) approach of teaching should be replaced by more application, modeling and real life problems; and b) diverse standards of measuring students' success in mathematics should be used ([22]). This is an example where an education system that was traditionally focused on development of students' procedural knowledge is recognizing the importance of development of students' conceptual understandings through use of rich application problems. In another example, as reported by Lien Her Bao in September of 2000 the ministry of education in Taiwan stated that focus in mathematics classroom should not be complicated algebraic manipulations as they can be replaced by using calculators and or computers ([23]). This is another international example where technology use is suggested to change the focus of instruction from procedural and rote activities to more conceptual ones. One of our main goals in this paper is to create technology based activities, as part of the dynamic course core that helps to develop teachers' conceptual knowledge. More mathematics concepts, as well as more challenging mathematics can be explored in depth by using technological tools in a mathematics classroom. These conceptual activities can also affect teachers' content knowledge for teaching and influence students learning of mathematics. ([18]). We assume that well implemented static and dynamic cores create such classroom discourse that allows students to build conceptual understanding of mathematics through representational fluency.

3 Technology for Developing Representational Fluency

For the past 25 years researchers have been discussing the role of technology in the mathematics classroom. They noted by using multiple representations, technology allows making connections and showing complex mathematical ideas that would be difficult to explore otherwise ([17]). Certain observations in mathematics would be very difficult for students without the help of technological tools. Using visual, graphical, numeric and geometric representation for abstract concepts makes mathematics more intuitive, interesting and accessible to a larger population of diverse learners. Geometrical interpretations of a problem introduced prior to algebraic interpretations can provide crucial intuition and motivation for learners grasping key concepts when solving a problem. The intuition can further assist teachers setting up mathematical conjectures and strategies of how to manipulate computer algebra systems (CAS) to verify their conjectures ([9]). Engaging with externalized representations through technology can allow students and teachers unique opportunities for exploration, discovery and revealing cognitive conflicts ([6]). Technology allows an access facilitated through a variety of technological capabilities to mathematical content that had not previously been included in the schools' curriculum. One of the most important technological capabilities is the power to provide accommodation for "multiple representations, hot-linked and interactive, and a key construct related to the availability of these representations is representational fluency" ([18]). Representational fluency referred to the interaction between representation and a learner, generally signifying learner's ability to move among various representations, drawing and transporting the meaning of a mathematical construct from one representation to another and gathering more knowledge about construct from every representation ([2,11,18]). Using multiple representations and developing representational fluency can support the development of conceptual knowledge and allow learners to relate procedural and conceptual knowledge ([5]). The activities below are designed to assist teachers to develop representational fluency in order to further improve their content knowledge for teaching of the aforementioned mathematical topics.

3.1 Memorizing a formula will only last for a test

It is natural to conjecture that if we introduced more geometric and graphic representations or motivations before involving complex algebraic manipulations, many students may have not lost their interests in mathematics so early. We hope evolving technological tools can assist pre-service teachers in learning adequate content in mathematics before they start teaching.

Confucius said 'Give a man a fish and you feed him for a day; teach a man to fish and you feed him for a lifetime'. We may say: Memorize a formula and you pass one exam; comprehend a formula and you discover more mathematics in a lifetime.

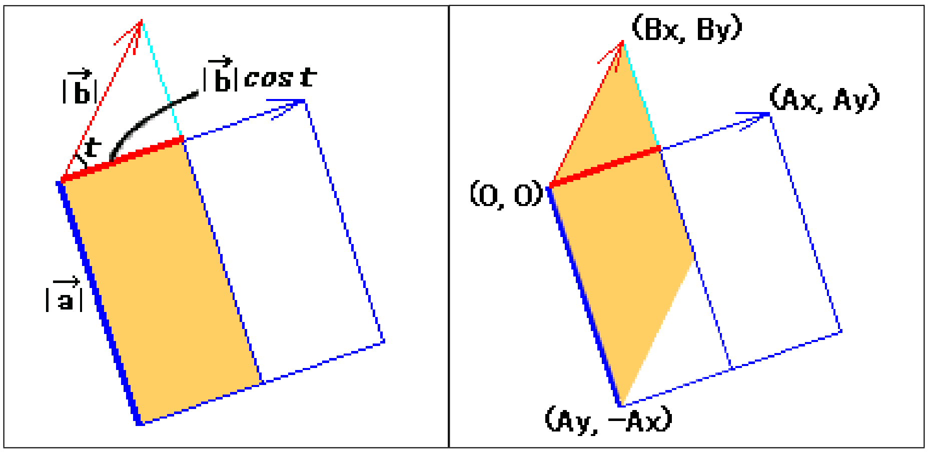
Many of us encountered the distance from a point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$

to be

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} \tag{1}$$

in high school. Authors recalled that we hardly cared how this formula was derived because there is no time to worry about the derivation during a 50 minute test. Instead of deriving this formula as part of static core, which can help to develop procedural knowledge, we present below one way technological tools can be used to help teachers to present this mathematical content focusing on conceptual knowledge development. This is an example of dynamic course core.

There are two important concepts in vector calculus, one is the dot product, $\overrightarrow{a} \cdot \overrightarrow{b}$, and the other is the cross product, $\overrightarrow{a} \times \overrightarrow{b}$,where $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors in R^n. However, most students would only remember the definition of $\overrightarrow{a} \cdot \overrightarrow{b}$ as $\sum a_i b_i$ or $|a|\ |b| \cos\theta$; or interpret this wrongly as the area of the parallelogram determined by $\overrightarrow{a}$ and $\overrightarrow{b}$ when $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors in R^2. Some students disagree that $\overrightarrow{a} \cdot \overrightarrow{b}$ represents the area of the parallelogram determined by $\overrightarrow{a}$ and $\overrightarrow{b}$, not because they understand completely the geometric interpretations of $\overrightarrow{a} \cdot \overrightarrow{b}$ and $\overrightarrow{a} \times \overrightarrow{b}$ respectively, but because they memorize the formula that the area of the parallelogram determined by $\overrightarrow{a}$ and $\overrightarrow{b}$ should be $\left\| \overrightarrow{a} \times \overrightarrow{b} \right\|$ but not $\overrightarrow{a} \cdot \overrightarrow{b}$. Thus we explore $\overrightarrow{a} \cdot \overrightarrow{b}$ and $\overrightarrow{a} \times \overrightarrow{b}$ in more detail. For two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ in 2D, $\overrightarrow{a} \cdot \overrightarrow{b}$ represents the area of the parallelogram determined by $\overrightarrow{a}$ and $\overrightarrow{b}_\perp$,where $\overrightarrow{b}_\perp$ is the vector that is perpendicular to $\overrightarrow{b}$, and with the same magnitude as $\overrightarrow{b}$. We may summarize this by observing the Figures 1(a) and 1(b), and encourage readers to explore this further by using the Java applet developed by IES of Japan ([19]).



Figures 1(a) and 1(b) Dot product

For two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ in 2D, it follows from the definition of cross product $\overrightarrow{a} \times \overrightarrow{b} = (\|a\|\ \|b\| \sin\theta)\ \overrightarrow{n}$, where $\overrightarrow{n}$ is a unit vector perpendicular to both $\overrightarrow{a}$ and $\overrightarrow{b}$ and whose direction is given by the right-hand rule, that the *magnitude* $\left\| \overrightarrow{a} \times \overrightarrow{b} \right\|$ *is equal to the area of the parallelogram determined by* $\overrightarrow{a}$ *and* $\overrightarrow{b}$. The java applet in [20] allows us to visualize this fact from a geometric point view.

Combining the ideas we explored on $\overrightarrow{a} \cdot \overrightarrow{b}$ and $\overrightarrow{a} \times \overrightarrow{b}$, it is not hard to understand why the volume of the parallelepiped determined by three vectors $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ in 3D can be written as

$$\left|\overrightarrow{a}\cdot\left(\overrightarrow{b}\times\overrightarrow{c}\right)\right| \tag{2}$$

$$= \|a\| \left\|\overrightarrow{b}\times\overrightarrow{c}\right\| |\cos\theta|, \tag{3}$$

where θ is the angle between the normal vector $\overrightarrow{n}$ of the plane determined by $\overrightarrow{b}$ and $\overrightarrow{c}$ and the vector $\overrightarrow{a}$. This can be further explored by using a Java applet (see [21]). Furthermore, the quantity $\|a\| (|\cos\theta|)$ *represents the height of the parallelepiped and it can also be used to find the distance from a point to a plane*, which we omit here.

Through exploration above, we discover that the volume of the parallelepiped determined by three vectors $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$, is actually equal to the area of a parallelogram determined by $\overrightarrow{a}$ and $\left(\overrightarrow{b}\times\overrightarrow{c}\right)_\perp$,which is a vector that is perpendicular to $\overrightarrow{b}\times\overrightarrow{c}$ and with the same magnitude as $\overrightarrow{b}\times\overrightarrow{c}$; we note that vector $\left(\overrightarrow{b}\times\overrightarrow{c}\right)_\perp$ can be any vector lying on the plane determined by $\overrightarrow{b}$ and $\overrightarrow{c}$. We 'discovered' an additional observation, which is not mentioned in a regular textbook. Thus, it is not hard to conjecture that more interesting mathematics concepts can be explored and discovered due to the advancement of technological tools. As part of dynamic course core this activity can allow teachers to create the type of classroom discourse that promotes students' develop of their conceptual understanding of mathematics. This activity can also help teachers to develop their own specialized content knowledge and horizon content knowledge.

We demonstrate below another example of a traditionally static core, Mean Value or Cauchy Mean Value Theorem, done differently. In this case, we are using geometric representation of traditional theorems. This makes the proof more interesting and intuitive.

Example 1 *About the proofs for the Mean Value or Cauchy Mean Value Theorems. These two theorems are very often used in applied mathematics. However, when it comes to the proof of either one of these theorems, it is not surprising that not many people can recall the proof.*

For example, the Mean Value Theorem can be stated below:

Suppose the function $f : [a, b] \to R$ is continuous on $[a, b]$ and differentiable on (a, b). Then there is a point x_0 in (a, b) at which

$$f'(x_0) = \frac{f(b) - f(a)}{b - a}.$$

To prove this theorem, in many traditional text books, one introduces the function h defined at each number x by the following equation:

$$h(x) = f(x) - \left(\frac{f(b) - f(a)}{b - a}\right)(x - a). \tag{4}$$

Then we use the fact that h satisfies the conditions for Rolle's theorem to deduce that there is a point c in (a, b) such that $h'(c) = 0$, and the Mean Value Theorem follows. However, we can inspire students to see how the function h is constructed from a geometric point of view. Suppose the blue curve (the darker curve) is given by $f(x) = \cos(x)$ and satisfies the conditions of the Mean Value theorem over the interval covering $(a, b) = (-\frac{\pi}{2}, 0.725)$ shown in Figure 2 below. We connect the line segment AB, where $A = (-\frac{\pi}{2}.0)$ and $B = (0.725, f(0.725))$ lying on $y = f(x)$ and ask the following question:

If we rotate line segment AB (while AB is attached to the graph of the function) *so that AB becomes a horizontal line segment, how would the graph of the original function appear?* We refer readers to the paper in [15], and explore that the curve in green or the lighter color is exactly the function $h(x)$ we are looking for by observing

$$\text{distance } EF = \text{distance } GD,$$

and we apply the Rolle's Theorem on $h(x)$.

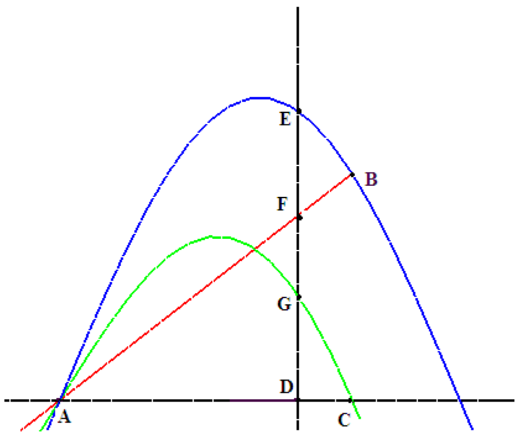


Figure 2. The graphs of two functions and a chord

We can prove the Cauchy Mean Value Theorem (CMV) in a similar manner. The statement of CMV can be seen below:

Suppose the function $f : [a, b] \to R$ and $g : [a, b] \to R$ are continuous and that their restrictions to (a, b) are differentiable. Moreover, assume that $g'(t) \neq 0$ for all t in (a, b). Then there is a point t in (a, b) at which

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(t)}{g'(t).}$$

We (again) see little geometric motivation of why we have two functions f and g and how the conclusion is obtained. In [15], we summarize how we can prove the CMV geometrically by extending the idea we described above when proving the MVT.

1. Assume functions f and g satisfy the condition of the Cauchy Mean Value Theorem, the Theorem holds, can be interpreted as any number t for which the parametric curve P defined by the equation

$$P(t) = [g(t), f(t)]$$

for $a \leq t \leq b$ has slope equal to the slope of the secant that runs from the point $(g(a), f(a))$ to the point $(g(b), f(b))$.

2. Equivalently, if we apply the Mean Value Theorem to the graph of a polar equation $r = h(t)$, by writing the polar equation in a parametric form

$$[x(t), y(t)] = [h(t)\cos(t), h(t)\sin(t)] = [g(t), f(t)], \tag{5}$$

we obtain the conclusion of Cauchy Mean Value Theorem (see [15] for details).

Through this example, we see that graphical and geometrical animations can make some uninspired proofs from traditional static content more accessible and interesting to more learners. It can also allow learners to gain additional insight to the meaning of mathematics by examining multiple representations. The Computer Algebra System (CAS) provides the thrust to the analytical proofs and makes mathematics challenging when students are inspired to investigate problems further**.**

3.2 Content evolve when technological tools advance

Part of specialized content knowledge is teachers' ability to present mathematics in the context that is relevant to their students and to make connection between theoretical mathematics and situations that appear "psychologically meaningful for students" ([5]). We further use the following two examples to demonstrate how theoretical mathematical ideas may be integrated in such 'psychologically meaningful' context. In these examples learners are presented with multiple representations of the mathematical topic, such as orthotomic and the caustic curves. This fits well with why the National Science Foundation of the USA is pushing the Science, Technology, Engineering and Mathematics (STEM) Program (see [24]). We need to integrate mathematics teaching with other applied disciplines. Students learn the concept of finding the inverse for a function, $f(x)$, in Pre-Calculus is to find a function $g(x)$ so that the graphs of $y = f(x)$ and $y = g(x)$ are symmetric to $y = x$. Many software including graphic calculators allow users to explore finding the inverse image of a parametric curve with respect to a slanted line $y = mx + b$, see Figure 3 below, that is described in [16]. Students can create many interesting pictures but one would wonder how the drawing mirror image with respect to a slanted line is related to mathematics.

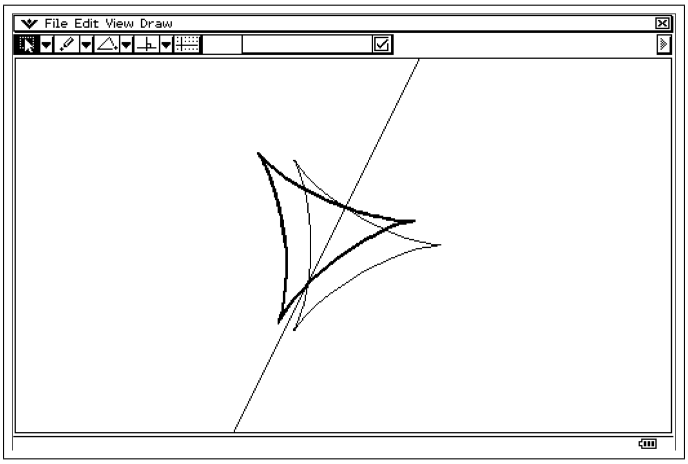


Figure 3. A reflection of Hypocycloid with respect to y=2x+1.

Example 2 *We extend the ideas of finding the inverse image of a parametric curve with respect to a foxed slanted line* $y = mx + b$ *to the idea of finding the reflection of a light source* B *on a curve* C_1 *with respect to a moving point* P *on* C_2 *(specifically with respect to a moving tangent line at the point* P *on* C_2*), which we call* B'*. The* ***locus of the point*** B' *is called the orthotomic curve and it is linked to the concept of a* ***caustic curve [link to a video clip].*** *The concepts can be extended to the corresponding concepts in 3-D.*

The complete description on this problem can be found in [16]. We shall see how technological tools can aid us to explore this complex ideass-in the area of Optics in Physics-with ease. We refer to Figure 4(a) below, where $C_1 = [x_1(t), y_1(t)] = [2\cos t - \cos 2t, 2\sin t - \sin 2t]$, $t \in [0, 2\pi]$ (shown as cardioid), and $C_2 = [s, f(s)] = [s, -2 + \sin(s)]$ (shown as a sinusoidal curve below). We pick a light source B on C_1, and set $C_3 = [p(t), q(t)]$ to be the reflection of C_1 with respect to the tangent line to C_2 at a point P. For a fixed point B on C_1 (light source at a point on C_1), the reflection of B on a curve C_1 with respect to a moving point P on C_2, which we call B', the **locus of the point** B' is called the *orthotomic curve.* Then we have the following observations:

1. The orthotomic curve of C_2 relative to B is shown in Figure 4(b) which can be experimented with by using a dynamic geometry software such as Geometry Expression ([26]) and verified by using a CAS such as Maple (See [27]).

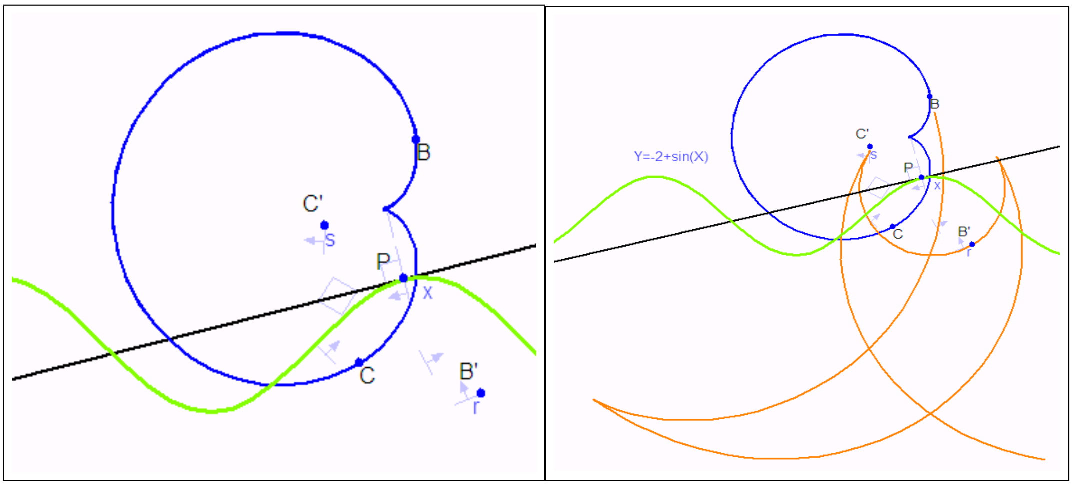


Figure 4(a) Original Curve and Figure 4(b) Original and its Orthotomic Curve

2. Picking another light source C on C_1 we obtain another orthotomic curve of C_2 relative to C (shown in black or darker curve in Figure 4(c)). We immediately discover the following observation by the 'dragging' mode with Geometry Expression. (see Figure 4(c)): *As* B *approaches* C*,the orange orthotomic curve (or the lighter curve) approaches the black orthotomic curve*

(the darker curve).

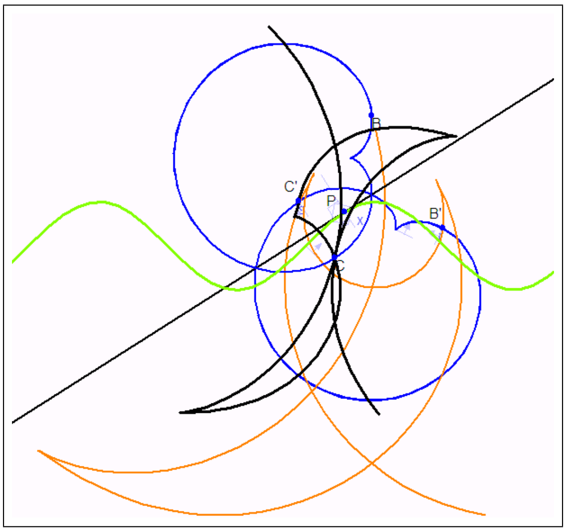


Figure 4(c) Two Orthotomic Curves

3. The sharp corner (cusp) of the black orthotomic occurs at the inflection point of C_2.

In the past, all of the observations above would have been difficult to be realized without a proper technological tool.

In Optics, we have often heard the term 'caustic curve'. This can be viewed as the envelope of rays reflected by a curve. We first note that the *evolute of a curve* C is the set of all its centers of curvature; it is equivalent to the envelope of all the normals to C. It can be shown that *the caustic generated by rays reflected by a curve C from a light source O (caustic of C relative to O) is equivalent to finding the evolute of the orthotomic of C relative to O. Or equivalently, the caustic curve is the centers of curvature of the family of orthotomic normals of a given curve relative to O.* We shall use a java applet (see [25]) to explore the relationship between the orthotomic and caustic curves, which are difficult concepts otherwise. We describe proposed activity as follows:

- Step 1: Create the original curve so that it roughly resembles an ellipse (see Figure 5):

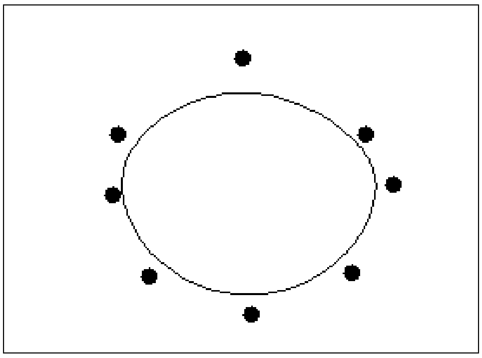


Figure 5. Creating a curve

- Step 2: Next, select the curves and click on 'Orthotomic Curve', the picture should look like Figure 6. Note the red center dot is the light source.

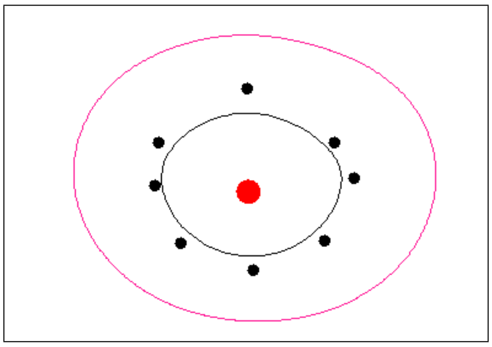


Figure 6. Original curve and its orthotomic curve

- Step 3: Now choose the Family of Orthotomic Normals, the picture should look like Figure 7:

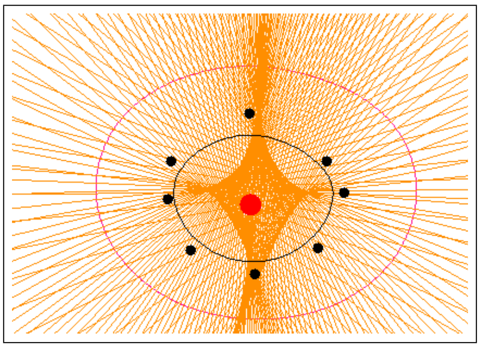


Figure 7. Original curve, its orthotomic and the evolute of the orthotomic

- Step 4: Click on the Caustics and note the following graph, we see that the caustic curve is the set of *centers of curvature of the family of orthotomic normals of a given curve relative to O* (the red or center dot).

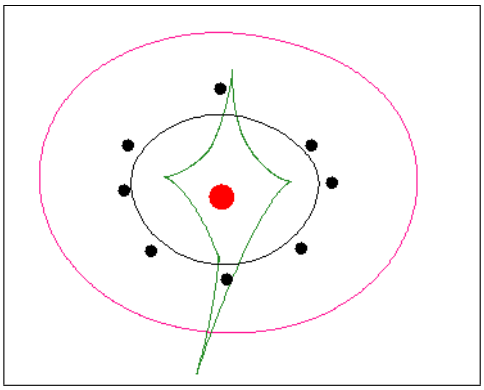


Figure 8. Original curve, its orthotomic and the caustic curve

We shall see these geometric observations in Figure 8 above are consistent with algebraic approaches, which can be verified when a computer algebra system such as Maple is used. For example, we consider the ellipse $[\frac{7}{5}\cos s, \frac{6}{5}\sin s]$, where $s \in [0, 2\pi]$, which simulates the black curve in Figure 8. The orthotomic curve of the ellipse relative to the origin, O, can be shown (see [16]) to be

$$\left[\begin{array}{c} -\sin(2\arctan(\dfrac{6\cos s}{7\sin s}))(-\frac{6}{5}\sin s - \dfrac{6\,(\cos(s)^2)}{5\sin s}) \\ -\cos\left(2\arctan\left(\dfrac{6\cos s}{7\sin s}\right)\right)\left(\frac{-6}{5}\sin s - \frac{6}{5}\dfrac{(\cos s)^2}{\sin s}\right) + \frac{6}{5}\sin s + \dfrac{6\,(\cos s)^2}{5\sin s} \end{array}\right], \tag{6}$$

which simulates the pink curve in Figure 8. We plot the original ellipse (in green or inner ellipse), its orthotomic curve (in blue or outer ellipse), and its caustic curve, can be shown to be the curve shown in red (or in the center of the inner ellipse), together in Figure 9.

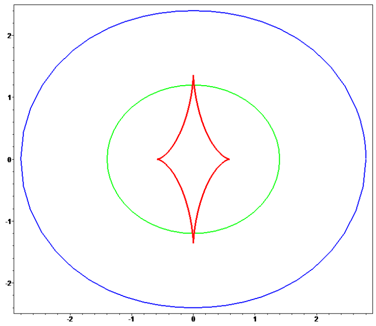


Figure 9. An ellipse, its orthotomic and caustic curves

We notice that three curves obtained in Figure 8 is indeed similar to corresponding curves described in Figure 9 and yet Figure 8 was obtained by exploring the concepts from a geometric point of view, which is accessible to anyone who would like to play with graphs, and yet Figure 9 is obtained after thorough theoretical and algebraic verifications. Obviously, geometric approaches provide critical intuition and motivations to learners and algebraic approaches challenge those students who want to do more. This example also shows that many complex topics in applied disciplines can be explored from a mathematical point of view when learners are empowered with proper knowledge both in subject matter and also in skills of manipulating latest technological tools.

Next we explore one example of finding the global minimum value of *total squared distances* among non-intersecting curves in the plane and surfaces in the space. The geometric interpretations will help learners appreciate the use of the Lagrange multipliers method and ideas learned from Linear Algebra.

Example 3 *We are given four convex surfaces in the space, represented by the orange, yellow, blue and purple surfaces (shown in Figure 10) which we will call S_1, S_2 S_3 and S_4 respectively. We want to find points A, B, C and D on S_1 (orange or the one at the left lower corner), S_2 (yellow or the one at the right lower corner), S_3 (blue or the one at the upper right) and S_4 (the purple or the one at the* *upper left) respectively so that the total distances $AB + AC + AD$ achieves its minimum.*

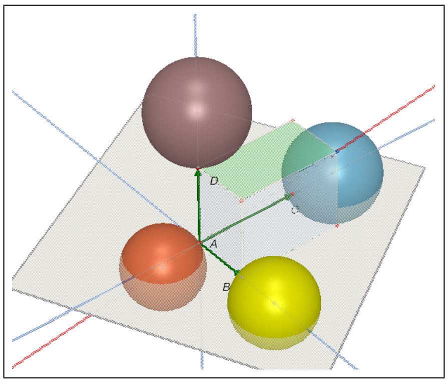
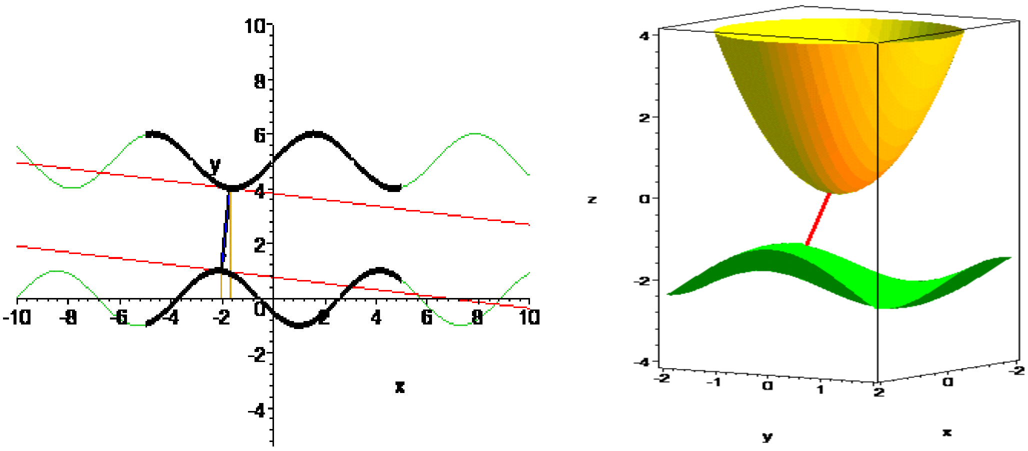


Figure 10. The shortest total squared distances among four convex surfaces

The details of this example can be found in [17]. This problem was started when we ask the following preliminary questions:

- If we are given two non-intersecting curves or surfaces in their respective domains, what is the shortest distance between these two curves and surfaces? With the exploration using a dynamic geometry software, it is not difficult to see the minimum distance occurs when the line segment connecting two points at two respective curves or surfaces is perpendicular to the each of the tangent lines or tangent planes at respective points. We demonstrate this by noting the following two Figures 11(a) and 11(b).



Figures 11 (a) and (b) Minimum distance between two curves and two surfaces.

- We extend the idea to finding the total shortest squared distances from one curve (say sinusoidal curve below) to two other curves-one is parabolic and the other one is a circle-by observing the following Figure 12. Does this diagram say anything about how we should position the normal vector at the point on the sinusoidal curve in relation to two other vectors? This will be clear later.

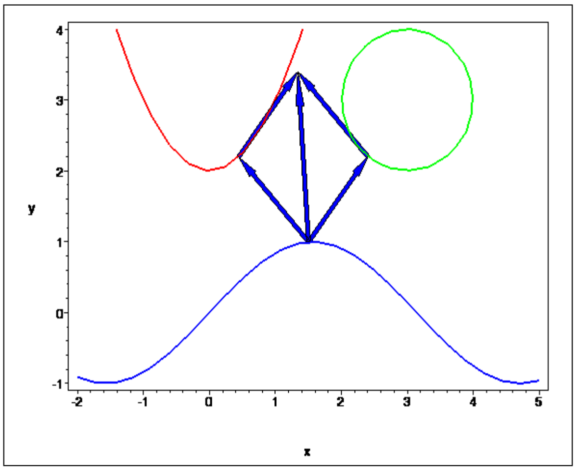


Figure 12. Shortest total squared distance from one curve to two other curves.

We note that the key observations for finding the shortest total squared distances $AB+AC+AD$, described in Figure 10, from a geometric point of view can be summarized as follows:

1. The vector AB should be parallel to the normal vector of the surface S_2 at B. This is equivalent to
$$AB = \lambda_2 \left(\nabla S_2 \text{ at } B\right) \text{ for some } \lambda_2. \tag{7}$$
2. The vector AC should be parallel to the normal vector of the surface S_3 at C. This is equivalent to
$$AC = \lambda_3 \left(\nabla S_3 \text{ at } C\right) \text{ for some } \lambda_3. \tag{8}$$
3. The vector AD should be parallel to the normal vector of the surface S_4 at D.This is equivalent to
$$AD = \lambda_4 \left(\nabla S_2 \text{ at } D\right) \text{ for some } \lambda_4. \tag{9}$$

To achieve the minimum distance for $AB + AC + AD$, we should also place point A so that the normal vector of S_1 at A is in the same direction of $AB + AC + AD$. This is equivalent to say we can find λ_1 so that

$$\lambda_1 \left(\nabla S_1 \text{ at } A\right) = \lambda_2 \left(\nabla S_2 \text{ at } B\right) + \lambda_3 \left(\nabla S_3 \text{ at } C\right) + \lambda_4 \left(\nabla S_2 \text{ at } D\right). \tag{10}$$

The following Theorem sums up what we discussed above, the proof can be found in [17].

Theorem 4 *If the total squared distances function*

$$f(\mathbf{x}_1, \mathbf{x}_2, , , , \mathbf{x}_p) = |\mathbf{x}_1 - \mathbf{x}_2|^2 + |\mathbf{x}_1 - \mathbf{x}_3|^2 + ... |\mathbf{x}_1 - \mathbf{x}_p|^2 \text{ has a global value,} \tag{11}$$

where $\mathbf{x}_i = (x_1^i, x_2^i, ..., x_n^i)$, $i = 1, 2, ...p$, *subject to* p *constraints*

$$g_1(\mathbf{x}_1) = c_1, g_2(\mathbf{x}_2) = c_2, ..., \text{ and } g_p(\mathbf{x}_p) = c_p, \tag{12}$$

at $\mathbf{x}_0 = (x_1^*, x_2^*, , , , x_p^*)$ *in its closed and bounded domain, then we can find coefficients,* $\lambda_j, j = 1, 2, ...p$, *so that*

$$\lambda_1 \nabla g_1(\mathbf{x}_0) = \sum_{j=2}^{p} \lambda_j \nabla g_j(\mathbf{x}_0). \tag{13}$$

This example further shows that there are plenty of interesting problems awaiting us to explore if learners are properly introduced to the latest technological tools and have an opportunity to engage with an appropriate dynamic content. It shows the importance for students to be able to integrate mathematics ideas they learned in Calculus and Linear Algebra. Therefore, when we consider mathematics reform, we need to consider hyperlinking the content properly. This example further demonstrates that geometrical interpretations of a problem will provide crucial intuition and motivation for students grasping key concepts when solving a problem. The intuition will further assist students setting up conjectures and strategies of how to manipulate a CAS timely and properly to verify their conjectures.

In this section we presented several examples of dynamic core which include technology based activities designed to: 1)improve learners' subject matter knowledge of aforementioned mathematical topics by using multiple representations; 2) increase teachers' specialized and horizon content knowledge of mathematics; and 3) show how mathematics can be taught in the context of apply disciplines.

4 Discussion

We proposed that dynamic core has an important role in developing preservice teachers' subject matter knowledge of mathematics. We used the bicycle mechanism, two wheels connected by a chain, as a metaphor to describe the connection between dynamic and static cores. Affect of the dynamic core can bring a type of change in mathematics classroom discourse that leads to learners' development of conceptual knowledge of mathematics, and preservice teachers' development of specialized content knowledge and knowledge at the mathematical horizon. In addition, this affect of dynamic core brings 'motion' to a static core that can be responsible for development of learners' procedural knowledge of mathematics and preservice teachers' common content knowledge of mathematics. On the other hand, the implementation of static course core can transform a classroom discourse in a way that might affect the quality of learners' engagement with dynamic core. As learners develop more procedural knowledge and teachers develop more common content knowledge, they can continue to acquire conceptual knowledge, SCK, and KMH.

In this paper, we presented three technology based mathematical conceptual activities, as part of our dynamic core, designed to improve teachers' mathematical knowledge for teaching the mean value theorem, the Cauchy mean value theorem, the inverse image of parametric curves, and the global minimum of total squared distances among non-intersecting curves. In the spring of 2009, the lead author implemented all three activities in his undergraduate mathematics course titled Topics in Mathematics-Mathematics and Technology. According to the data gathered from the course evaluation, many preservice teachers enjoyed this technology based class. As a result, they built conceptual knowledge of the corresponding mathematical ideas. Based on classroom observations, these activities help to develop learners' representational fluency of aforementioned topics and deepen their conceptual understanding of these mathematical topics. Since it is challenging to measure procedural and conceptual knowledge directly, an analysis can be based on assessment of relationship between procedural and conceptual tasks representing these knowledge types ([7]). Therefore it is important to continue development and implementation of the types of activities described in this paper.

Providing adequate subject matter knowledge and inspiring creative thinking skills are pivotal in a mathematics curriculum. Technology cannot solve all our problems but can assist us in achieving the balance between an amount and type of mathematics knowledge that learners need to acquire to be successful mathematics teachers. Implementing technological tools into teaching and learning is not a trivial task and it will be an ongoing pedagogical issue for many years to come. It is imperative, in our view, to build a curriculum where teachers need to know when and how to introduce a subject with lots of mathematical intuition and motivations so mathematics is more accessible and interesting to more students at younger ages. Also, technological tools can be powerful in developing teachers' own conceptual understandings of mathematics as well as their mathematical knowledge for teaching mathematics.

References

[1] Ball, D.L., Thames, M.H., & Phelps, G. (2009). Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5), 389-407.

[2] Becker, J. R., & Rivera, F. (n.d.). 'Establishing and justifying algebraic generalization at the sixth grade level'. Retrieved January 25, 2010 from 130.65.82.235/rivera/nsfcareer.fa05/RR_becker.rtf

[3] Borwein, J.M. . (2005) 'The experimental mathematician: the pleasure to discover and the role of proof'. International Journal of Computers for Mathematical learning, 10, 75-108.

[4] Borwein, J. M. & Bailey, D. H. (2003) Mathematics by experiment: plausible Reasoning in 21st Century, Natic, MA: AK Peters.

[5] Haapasalo, L. (2008). Perspectives on Instrumental Orchestration and Assessment - from Challenges to Opportunities. ATCM 2008 Conference Proceedings. Bangkok, Thailand, December 2008.

[6] Haapasalo, L. (2007) 'Adaptaing mathematics education to the needs of ICT'. The Electronic Journal of Mathematics and Technology, Volume 3, Number 1, ISSN 1933-2823.

[7] Haapasalo, L. & Kadijevich, Dj. (2000). Two Types of Mathematical Knowledge and Their Relation. Journal für Mathematik-Didaktik, 21, 2, 139-157.

[8] Heid, M.K. (1997) 'The technological revolution and the reform of school mathematics'. American journal of education, 106(1), 5-61.

[9] Hill, H., Ball, D. L., & Schilling, S. (2008) 'Unpacking "pedagogical content knowledge": Conceptualizing and measuring teachers' topic-specific knowledge of students'. Journal for Research in Mathematics Education, 39 (4), 372-400.

[10] Manizade, A. G., & Lundquist, B. (2009) 'Learning about proof by Building Conjectures'. PMENA 2009 Conference Proceedings. Atlanta, Georgia, September 2009.

[11] Rittle-Johnson, Bethany; Alibali, Martha Wagner. 'Conceptual and procedural knowledge of mathematics: Does one lead to the other?', Journal of Educational Psychology. 91(1), Mar 1999, 175-189.

[12] Sandoval, W.A., Bell, P., Coleman, E., Enyedy, N., & Suthers, D. (2000) 'Designing knowledge representations for epistemic practices in science learning'. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

[13] Shulman, L. S. (1986) 'Paradigms and research programs in the study of teaching.' In M.C. Wittrock (Ed.), Handbook of research on teaching. New York: MacMillan.

[14] Shulman, L. S. (1986) 'Those who understand: Knowledge growth in teaching'. Educational Researcher Feb. 1986: 4-14. (AERA Presidential Address).

[15] Yang, W.-C. (2007) 'Revisit Mean Value, Cauchy Mean Value and Lagrange Remainder Theorems', The Electronic Journal of Mathematics and Technology, Volume 1, Issue 2, ISSN 1933-2823, Mathematics and Technology, LLC, USA.

[16] Yang, W.-C and Lo, M.-L. (2008) 'General Inverses in 2-D, 3-D, applications inspired by Technology'. The Electronic Journal of Mathematics and Technology, Volume 2, Issue 3, ISSN 1933-2823, Mathematics and Technology, LLC, USA.

[17] Yang, W.-C. (2009) 'Some Geometric Interpretations of The Total Distances Among Curves and Surfaces'. The Electronic Journal of Mathematics and Technology, Volume 3, Issue 1, ISSN 1933-2823. Mathematics and Technology, LLC, USA.

[18] Zdiek, R.M.; Heid, M. K.; Blume, G. W.; Dick, T. P. (2007) 'Research on Technology in Mathematics Education: A Perspective of Constructs', Second Handbook of Research on Mathematics Teaching and Learning. P. 1169-1207.

[19] A Java applet on dot product, International Education Software of Japan, http://www.ies.co.jp/math/java/.

[20] A Java applet on cross product, http://www.math.umn.edu/~nykamp/m2374/readings/crossprod/.

[21] A Java applet on parallelepiped, http://www-math.mit.edu/18.013A/HTML/chapter04/section01.html.

[22] Beijing Youth Daily, March 7, 2001, a daily news paper in China.

[23] Lien Her Bao, September 4, 2000, a daily news paper in Taiwan.

[24] National Science Foundation in the USA and STEM Program, http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5488.

[25] Yoshizawa, S., '*Curve Simulator*', hosted by the Electronic Journal of Mathematics and Technology (eJMT), https://php.radford.edu/~ejmt/Resources/CurveSimulator/CurveSimulator.html.

[26] Geometry Expressions 1.0.55, a product of Saltire Software, 2006, http://www.geometryexpressions.com/.

[27] Maple, a product of Maplesoft, http://www.maplesoft.com/.